REPORT No. 173.

RELIABLE FORMULAE FOR ESTIMATING AIRPLANE PERFORMANCE AND THE EFFECTS OF CHANGES IN WEIGHT, WING AREA, OR POWER.

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SUMMARY.

This paper, which was prepared for publication by the National Advisory Committee for Aeronautics, contains the derivation and the verification of formulae for predicting the speed range ratio, the initial rate of climb, and the absolute ceiling of an airplane. It is shown that the ratio of the maximum speed V_M to the minimum speed V_S is given by

$$\frac{V_M}{V_S}=\frac{K_1\,\eta_m^{1/3}}{\left(V_S\cdot\frac{W}{HP}\right)^{1/3}}$$

where η_m is the maximum propeller efficiency and K_1 is a constant with an average value of 20.30 when V is in M. P. H. and $\frac{W}{HP}$ is in lb./BHP.

The rate of climb at sea level, C_o, is given by

$$C_o=33000\left(\frac{K_2\,\eta_m}{\frac{W}{HP}}-\frac{(2V_S+V_M)}{1125\left(\frac{L}{D}\right)}\right)$$

where $\left(\frac{L}{D}\right)$ is the overall value for the airplane at the angle for best climb (maximum value of $\frac{L}{D}$ is to be used) and K_2 is a constant found to be

$$K_2=\left(\frac{V_M}{V_S}\right)^{-0.27}$$

The absolute ceiling is given indirectly by

$$\frac{HP_{ao}}{HP_{ro}}=\frac{K_4\left(\frac{L}{D}\right)}{\left(\frac{1}{\eta_m}\cdot V_S\cdot\frac{W}{HP}\right)^{0.80}}$$

K_4 having an average value of 61.7 when V_S is in M. P. H. and $\frac{W}{HP}$ is in lb./BHP. The absolute ceiling is obtained by reference to the usual curves of absolute ceiling against the ratio $\frac{HP_{ao}}{HP_{ro}}$. These curves are given in National Advisory Committee for Aeronautics Report No. 171.

Standard formulae for service ceiling, time of climb, cruising range, and endurance are also given in the conventional forms.

INTRODUCTION.

It is of the greatest importance that the aeronautical engineer be able to predict with considerable accuracy the effect of changes in weight and power on the performance of an airplane. The usual procedure has been in accordance with that outlined in Bairstow's Applied Aerodynamics, Chapter IX; that is, the performance is read from a series of empirical curves based on test data. This method at times gives good results, but it can not be depended on when the variations in either wing loading or power loading are great. Warner, in an article on "Airplane performance formulas," S. A. E. Journal, June, 1922 (vol. 10, No. 6), develops some very interesting formulae which appear in general to give better results than the empirical curves previously mentioned.

The formulae for speed range, rate of climb, and absolute ceiling, which are derived in this paper, were developed in the Bureau of Aeronautics of the Navy Department by the writer in an attempt to place performance prediction on a more sound basis. The formulae have been used in routine work for over a year with gratifying results, particularly in case of the formulae for speed range and rate of climb. The formula for absolute ceiling has just been developed and has not been given a thorough verification, but it appears to fulfill the requirements for accurate work, especially when it is desired to calculate the effect of changes in $\frac{W}{S}$ and $\frac{W}{HP}$.

The formulae for service ceiling, time of climb, cruising radius, and endurance are given in the well-known forms and require no comment. It is considered that their derivation may be of interest at this time.

DERIVATION OF SPEED RANGE FORMULA.

If the lift of the body, tail, and minor parts of an airplane be neglected, the speed in horizontal flight must be given by the fundamental equation

$$W = C_L \frac{\rho}{2} S V^2 \tag{1}$$

and at standard density

$$V = \frac{K}{\sqrt{C_L}} \tag{1a}$$

The stalling speed V_S corresponds to the maximum lift coefficient C_{LM}:

$$V_S = \frac{K}{\sqrt{C_{LM}}} \tag{1b}$$

Dividing (1a) by (1b)

$$\frac{V}{V_S} = \sqrt{\frac{C_{LM}}{C_L}} \tag{2}$$

Referring to the plot of C_L, C_D, and $\frac{L}{D}$ against angle of attack for any standard airfoil, it will be seen that the slope of the lift curve is substantially constant from zero lift to a value approximately 90 per cent of the maximum. It will also be noted that owing to the small change in drag coefficient with angle at low values of C_L, the slope of the $\frac{L}{D}$ curve is likewise substantially constant from $C_L=0$ to $C_L=.40\ C_{LM}$. That is, $\frac{L}{D}$ may be written proportional to C_L

$$\frac{L}{D} = M \cdot C_L = N \left(\frac{L}{D}\right)_{MAX} \cdot L_L \tag{3}$$

substituting this in equation (2)

$$\frac{L}{D} = K \left(\frac{V}{V_S}\right)^{-2} = K \left(\frac{L}{D}\right)_M \cdot \left(\frac{V}{V_S}\right)^{-2} \tag{4}$$

The power required for horizontal flight is

$$THP=\eta\cdot BHP=\frac{DV}{375} \tag{5}$$

where D is the drag in lb. and V the velocity in M. P. H. Since $W=L$ in horizontal flight

$$D=\frac{W}{\left(\frac{L}{D}\right)} \text{ and}$$

equation (5) may be written

$$\eta\cdot BHP=\frac{WV}{375\left(\frac{L}{D}\right)} \tag{5a}$$

at maximum speed $V=V_M$ so that

$$\frac{V_M}{\left(\frac{L}{D}\right)}=\frac{375\eta}{\left(\frac{W}{BHP}\right)} \tag{5b}$$

Substituting in equation (5b) the value of $\left(\frac{L}{D}\right)$ from equation (4)

$$\frac{V_M^3}{V_S^2}=\frac{K\cdot\eta\left(\frac{L}{D}\right)_M}{\left(\frac{W}{BHP}\right)} \tag{6}$$

dividing by V_S

$$\left(\frac{V_M}{V_S}\right)^3=\frac{K\cdot\eta\left(\frac{L}{D}\right)_M}{V_S\cdot\left(\frac{W}{BHP}\right)}$$

or

$$\frac{V_M}{V_S}=\frac{K\sqrt[3]{\eta}\left(\frac{L}{D}\right)_M^{\frac{1}{3}}}{\sqrt[3]{V_S\left(\frac{W}{BHP}\right)}} \tag{7}$$

This speed range formula holds true for all values of $\left(\frac{V_M}{V_S}\right)$ greater than 1.60, the practical limit to the validity of equation (3). In order to demonstrate this point the values of C_{LM} and the range in C_L over which $\left(\frac{L}{D}\right)$ is proportional to C_L have been compiled for a series of well-known airfoil sections and are given in Table I.

Equation (7) was derived from a consideration of the characteristics curves of airfoils. It applies with even more exactness to airplanes, since at high speeds the parasite drag coefficient is practically constant and fully as large as the wing drag coefficient in practically all cases and greater in many cases. The effect of variations in wing drag coefficient will therefore be reduced.

It should be noted that at any given density $\frac{V_M}{V_S}$ depends only on the corresponding lift coefficients. At any altitude the correct value of $\frac{V_M}{V_S}$ is obtained from equation (7) by using the proper values of V_S, η, and $\frac{W}{HP}$ corresponding to the stalling speed, propeller efficiency, and engine power at this altitude.

PERFORMANCE CALCULATIONS.

In order to verify the speed range formula and to obtain data for a further study of the effect of changes in wing loading and power loading, routine performance calculations have been made for a hypothetical airplane loaded and powered to the 30 conditions represented by the combinations of five wing loadings with six power loadings. In these calculations the airplane is assumed unchanged except for weight and power, so that the results represent the true effect of variables studied.

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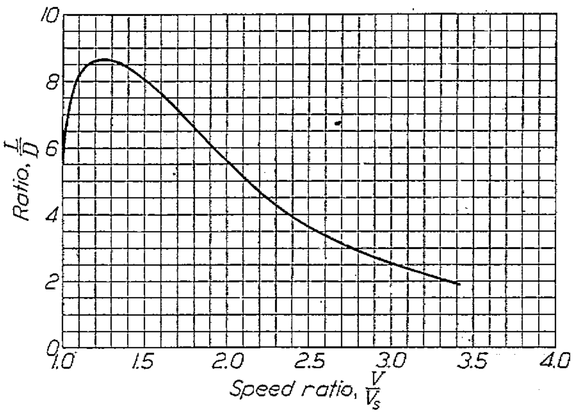

FIG. 1.—Variation of $\frac{L}{D}$ with $\frac{V}{V_s}$

The data given in Table II and Fig. 1 are obtained from wind tunnel test data on a complete model of an airplane which had approximately 300 square feet of wing area. These data have been corrected to the proper elevator setting required at each angle of attack for an airplane with 300 square feet of wing area of R. A. F.–15 section. Table III contains the faired values of $\frac{L}{D}$ vs. $\frac{V}{V_s}$ from the curve of Fig. 1. These values are used to calculate the curves of power required, HP_r, for each wing loading. At this point, it is to be noted that no allowance is made for the slipstream effect, chiefly because of the simplification entailed.

The method of calculating both power required, HP_r, and power available, HP_a, is exemplified in Table IV. It is assumed that the normal R. P. M. is 1,800 at high speed, decreasing uniformly to 1,600 at low speed. This decrease in R. P. M. is perhaps slightly more than that usually obtained when the speed range is low, although it is a fair average. For this reason the rate of climb and ceiling values for the low-powered cases will be found slightly low. The propeller diameter is calculated by means of the common nomograms to absorb the required BHP at 1,800 R. P. M. at the normal high speed. Two slight errors enter here; the high speed assumed was not in every case the actual high speed, and the nomogram does not give the true diameter—the average error is about 0.10 foot. These errors are quite inconsequential, however.

Propeller efficiencies are obtained from the curves of National Advisory Committee for Aeronautics Report No. 168. The maximum efficiency is determined by the $\frac{V}{ND}$ at high speed, and the efficiency at any other $\frac{V}{ND}$ is given in terms of the maximum efficiency by the "general efficiency curve."

It is assumed that the BHP is directly proportional to N over the range involved in each case. This assumption is justified by the power curves of modern engines, provided that N is not too high.

Tables IV to VIII, inclusive, give HP_r for wing loadings of 4, 6, 8, 10, and 14 lb./sq. ft. and HP_a for power loadings of 6, 8, 11, 16, 20, and 24 lb./HP at each wing loading, as calculated by the method just outlined. These data are plotted on Figs. 2 to 6, inclusive. The essential performance data from these plots is given in Tables IX to XIII, inclusive.

VERIFICATION OF SPEED RANGE FORMULA.

The value of K_1 in the speed range formula, equation (7), is determined for each of the 30 combinations of $\frac{W}{S}$ and $\frac{W}{HP}$ in Tables IX to XIII. It will be noted that so long as $\frac{V_M}{V_s}$ is greater than 1.70, K is substantially constant with an average value of 20.3. The average deviation from this value over the range for which the formula holds true is less than 1 per cent. The accuracy in determining V_M is probably of the order of 1 per cent, so that the formula is verified.

The values of K_1 have also been determined from reliable performance data for a number of well-known airplanes, which are given in Table XIV. It appears that for a normal airplane the value of K_1 varies not more than 5 per cent from the average value of 20.30 previously determined. The extreme variation in K_1 noted for the F–5–L seaplane is probably due more to the low speed range than to any other cause, although the value of $\left(\frac{L}{D}\right)$ max is known to be much below the average.

COMMENT ON SPEED RANGE FORMULA.

If the speed range $\left(\frac{V_M}{V_S}\right)$ be plotted logarithmically against the power loading $\left(\frac{W}{HP}\right)$, it is found that

$$\left(\frac{V_M}{V_S}\right) \propto \left(\frac{W}{HP}\right)^{-0.36} \tag{8}$$

since it may be shown in the same manner that

$$\left(\frac{V_M}{V_S}\right) \propto \left(\frac{W}{\eta \cdot BHP}\right)^{-0.33_3} \tag{9}$$

it is to be concluded that

$$\eta_m \propto \left(\frac{W}{HP}\right)^{0.027} \tag{10}$$

for the particular case in which $N = 1{,}800$ R. P. M. This relation simplifies the calculation when η_m is unknown. The speed range formula may then be written

$$\frac{V_M}{V_S} = \frac{19.90}{V_S^{.33}\left(\frac{W}{HP}\right)^{0.36}} \tag{11}$$

to be used when the maximum efficiency is unknown. It will be found more satisfactory, however, to use the complete formula, equation (7), when η_m is known. The value of K is obviously variable with the type of airplane. It is recommended that for the average airplane of clean design K be taken equal to 20.3. The figure will probably vary from 19.5 to 21.0 according to the design, but it requires an unusually clean design and high-speed range to secure values of K in excess of 20.5.

The formula may be used to determine the effect of changes in weight or power of an airplane of known performance with great accuracy. This is, the true value of K may be determined from the known performance and used with the new value of V_S and $\left(\frac{W}{HP}\right)$.

The V_S in this formula is the stalling speed. It is obviously very important to use the correct value, which is given by the well-known equation

$$V = \sqrt{\frac{W}{C_{LM}\frac{\rho}{2}S}} \tag{12}$$

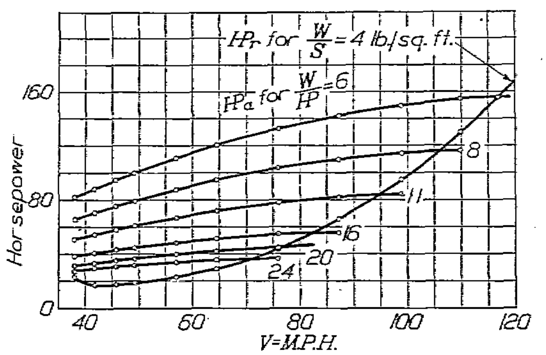

FIG. 2.—Power curves for $\frac{W}{S} = 4$ *lb./sq. ft.*

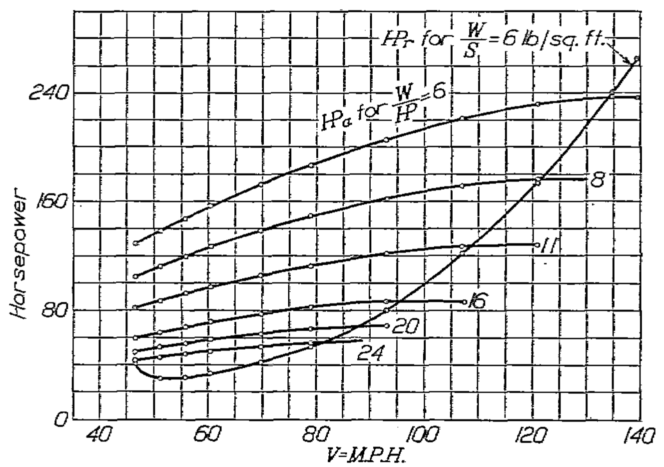

FIG. 3.—Power curves for $\frac{W}{S} = 6$ *lb./sq. ft.*

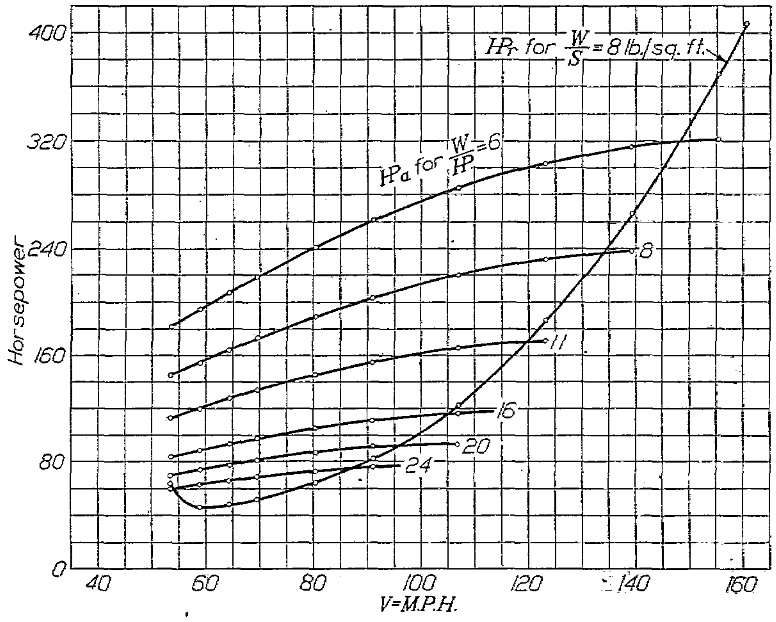

FIG. 4.—Power curves for $\frac{W}{S}$ = 8 lb./sq. ft.

where C_{LM} is the maximum lift coefficient of the *wings* for the particular arrangement used. At sea level and for V_S in M. P. H., equation (12) reduces to

$$V_S = 19.8\sqrt{\frac{\left(\frac{W}{S}\right)}{C_{LM}}} \qquad (12a)$$

which may be solved by a single setting on a slide rule.

DERIVATION OF FORMULA FOR INITIAL RATE OF CLIMB.

The maximum rate of climb at sea level will correspond to the greatest excess horsepower, or difference between power available and power required. The power available is

$$HP_a = K_2 \cdot \eta_m \cdot HP \qquad (13)$$

where K_2 is some constant depending on the engine and propeller combination. The power required is

$$HP_r = \frac{D V_C}{375} = \frac{W \cdot V_C}{375\left(\frac{L}{D}\right)} \qquad (14)$$

where V_C is the airspeed for best climb $\left(\frac{L}{D}\right)$ and the overall value for the airplane. Table XV contains a study of V_C with relation to V_S and V_M as given by the data in Tables IX to XIII, inclusive. It is shown in Table XV that for all practical purposes the best climbing speed, V_C, *at sea level*, is greater than the stalling speed, V_S, by one-third of the difference between the maximum speed, V_M, and the stalling speed, V_S. That is

$$V_C = V_S + \frac{1}{3}(V_M - V_S) = \frac{(2V_S + V_M)}{3} \qquad (15)$$

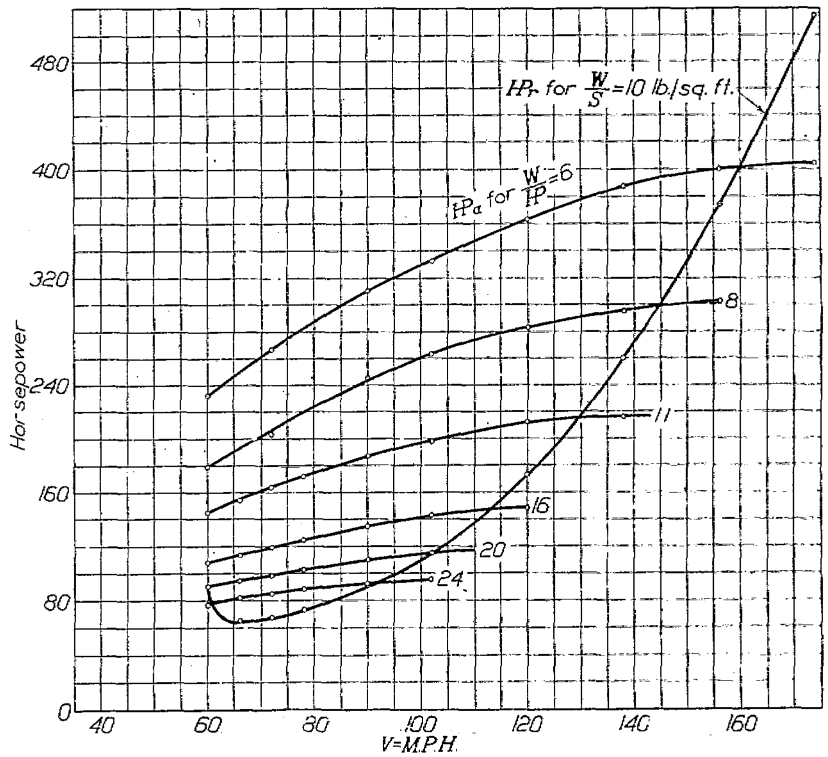

FIG. 5.—Power curves for $\frac{W}{S}$ = 10 lb./sq. ft.

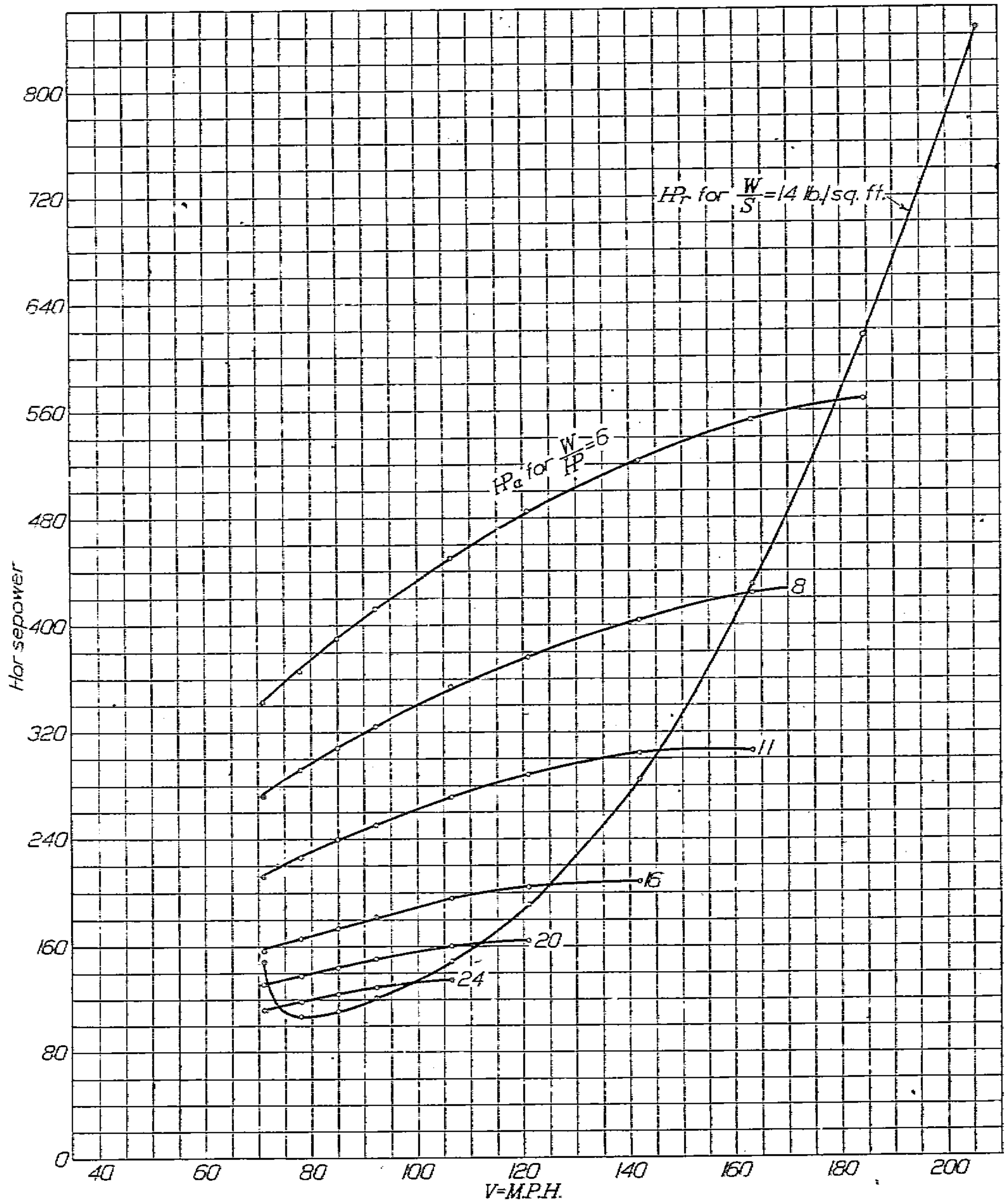

FIG. 6.—Power curves for $\frac{W}{S}=14$ lb./sq. ft.

substituting (15) into equation (14) gives

$$HP_r=\frac{W(2V_S+V_M)}{1125\left(\frac{L}{D}\right)} \tag{14a}$$

The initial rate of climb in feet per minute is, therefore,

$$\begin{aligned} C_o&=\frac{33000}{W}(HP_a-HP_r)\\ &=\frac{33000}{W}\left\{(K_2\eta_m HP)-\frac{W(2V_S+V_M)}{1125\left(\frac{L}{D}\right)}\right\}\\ &=33000\left\{\frac{K_2\eta_m}{\left(\frac{W}{HP}\right)}-\frac{(2V_S+V_M)}{1125\left(\frac{L}{D}\right)}\right\} \end{aligned} \tag{16}$$

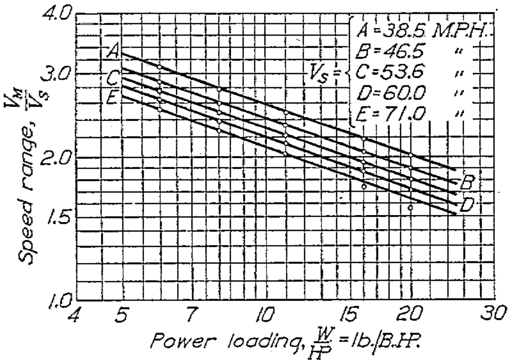

FIG. 7.—Variation of speed range $\frac{V_m}{V_s}$ with power loading, $\frac{W}{HP}$ $\left(\frac{V_m}{V_s}\right) \propto \left(\frac{W}{HP}\right)^{-0.36}$

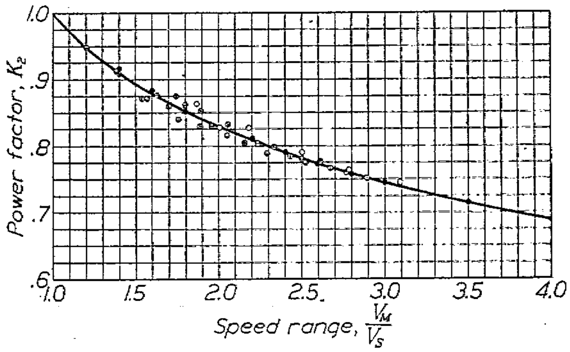

FIG. 8.—Variation of constant K_2 in formula for initial rate of climb. $C_0=33000\left[\frac{K_2\eta m}{\left(\frac{W}{HP}\right)}-\frac{2V_s+V_M}{1125\left(\frac{L}{D}\right)}\right]$ $K_2=\left(\frac{V_M}{V_s}\right)^{-0.27}$

The value of the constant K_2 is yet to be determined.

Table XVI contains calculations for K_2 using data from Tables IX to XIII, inclusive. As expected, K_2 decreases with increase in the speed range $\left(\frac{V_M}{V_S}\right)$. Plotting K_2 against $\left(\frac{V_M}{V_S}\right)$ as in Fig. 8, it is found that the points fall on or near to a smooth curve which has the equation

$$K_2=\left(\frac{V_M}{V_S}\right)^{-0.27} \tag{17}$$

as shown by the logarithmic plotting of the same data on Fig. 9.

In using the formula for initial climb, equation (16), the proper value of K_2 must be used. This value may either be read from Figs. (8) or (9) or calculated from equation (17). It will be found that for very low initial rates of climb the formula is unreliable, since small percentage errors in either member of equation (16) under these conditions may mean large percentage errors in C_0. The limiting value of C_0 is usually about 400 ft./min.

Obviously there is another unknown in this equation, the overall $\left(\frac{L}{D}\right)$ for the airplane.

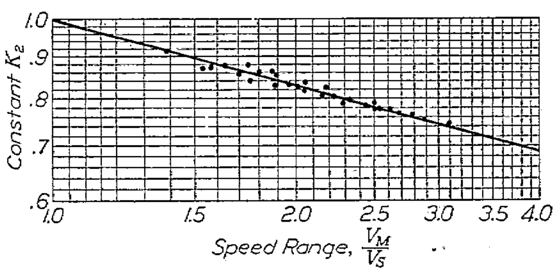

FIG. 9.—Variation of constant K_2 in formula for initial rate of climb. $C_0=33000\left[\frac{K_2\eta m}{\left(\frac{W}{HP}\right)}-\frac{(2V_s+V_M)}{1125\left(\frac{L}{D}\right)}\right]$ $K_2=\left(\frac{V_M}{V_s}\right)^{-0.27}$

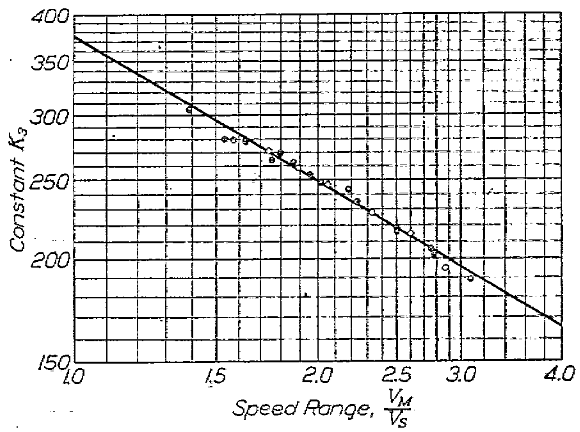

FIG. 10.—Variation of constant K_3 with $\left(\frac{V_M}{V_s}\right)$ $\frac{HP_{ao}}{HP_{ro}}=\frac{K_3\eta m\left(\frac{L}{D}\right)}{V_s\left(\frac{W}{HP}\right)}$ $K_3=375\left(\frac{V_M}{V_s}\right)^{-0.60}$

In most cases this is known to the accuracy required in C_o. When $\left(\frac{L}{D}\right)$ is unknown, the following values will be found fairly representative for the general types:

Unusually clean designs (monoplanes)	8.5–9.5
Clean designs (average 8.0)	7.5–8.5
Mediocre designs (excessive parasite resistance)	6.5–7.5

Fortunately the $\frac{L}{D}$ curve for the entire airplane is quite flat near the maximum value, so that little error is introduced by the use of maximum instead of the actual $\left(\frac{L}{D}\right)$. In most cases the value $\frac{L}{D}=8.0$ gives sufficiently accurate results, as shown by Table XVII, where observed performance data is used to check formula (16).

DERIVATION OF FORMULA FOR ABSOLUTE CEILING.

The absolute ceiling is dependent upon the ratio HP_{ao}/HP_{ro}, which is easily calculated. Dividing equation (13) by equation (14) and substituting V_S for V_C gives

$$\frac{HP_{ao}}{HP_{ro}}=\frac{\dfrac{K_3\cdot\eta_m\cdot HP}{W\cdot V_S}}{375\left(\dfrac{L}{D}\right)}$$

$$=\frac{K_3\cdot\eta_m\left(\dfrac{L}{D}\right)}{V_S\cdot\left(\dfrac{W}{HP}\right)} \tag{18}$$

Table XVIII contains calculations for K_3, based on the data in Tables IX to XIII, inclusive. The values of K_3 so obtained are then plotted against $\left(\frac{V_M}{V_S}\right)$ in Fig. 10, from which it appears that

$$K_3=375\left(\frac{V_M}{V_S}\right)^{-0.60} \tag{19}$$

From equation (7)

$$\left(\frac{V_M}{V_S}\right)^{-0.60}=\left\{\frac{20.3\eta_m^{1/3}}{\left(V_S\cdot\dfrac{W}{HP}\right)^{1/3}}\right\}^{-0.60}=\frac{1}{6.09}\left(\frac{1}{\eta_m}\cdot V_S\cdot\frac{W}{HP}\right)^{0.20} \tag{20}$$

From (19) and (20)

$$K_3=K_4\left(\frac{1}{\eta_m}\cdot V_S\cdot\frac{W}{HP}\right)^{0.20}=61.7\left(\frac{1}{\eta_m}\cdot V_S\cdot\frac{W}{HP}\right)^{0.20} \tag{21}$$

Substituting (21) into (18) gives

$$\frac{HP_{ao}}{HP_{ro}}=\frac{K_4\left(\dfrac{L}{D}\right)}{\left(\dfrac{1}{\eta_m}\cdot V_S\cdot\dfrac{W}{HP}\right)^{0.80}} \tag{22}$$

which is the formula for absolute ceiling, to be used in conjunction with a curve of absolute ceiling vs $\frac{HP_{ao}}{HP_{ro}}$.

Table XIX contains calculations for K_4 based on the data in Tables IX to XIII. The extreme variation is from 60.2 to 63.2, with an average value of $K_4=61.7$. This is the same value obtained by direct calculation in (21), where $K_4=375\div(20.3)^{-0.60}=61.7$.

Table XX contains a comparison of actual absolute ceilings for a series of well-known airplanes with the values calculated by equation (22), using the curve of HP_{ao}/HP_{ro} given in National Advisory Committee for Aeronautics Report No. 171. The agreement, in general, is quite satisfactory, considering that a constant value $\frac{L}{D}=8$ was used for each case.

SERVICE CEILING.

The service ceiling is defined as the altitude at which the rate of climb is 100 ft./min. From the results of climb tests it is found that the rate of climb decreases uniformly with altitude from a maximum at sea level to zero at the absolute ceiling. That is, at any altitude the rate of climb is given by the equation

$$C = C_o - C_o \frac{y}{H_a} \tag{23}$$

Where H_a is the absolute ceiling, C_o the initial rate of climb, and C the rate of climb at altitude y. At the service ceiling $C=100$ and

$$\frac{y}{H_a} = \frac{C_o - 100}{C_o}$$

$$y = H_S = H_a \frac{C_o - 100}{C_o} \tag{24}$$

where H_S is the service ceiling.

TIME OF CLIMB.

The time to climb to any altitude, based on the assumption of a uniform decrease in rate of climb with altitude, may be found in any good treatise on airplane performance. The derivation of the equation may be of interest.

Since the rate of climb $C=\frac{dy}{dt}$ equation (23) may be written

$$\frac{dy}{dt} = C_o\left(1 - \frac{y}{H}\right)$$

or

$$\frac{dy}{(H-y)} = \frac{C_o}{H}dt \tag{23a}$$

Integrating

$$\log_e (H-y) = -\frac{C_o}{H}t + C$$

when $t=0$, $y=0$, and $C=log_e\ H$

therefore,

$$-\frac{C_o}{H}t = \log_e\left(1 - \frac{y}{H}\right)$$

or

$$e^{-\frac{C_o}{H}t} = 1 - \frac{y}{H}$$

solving for y

$$y = H\left(1 - e^{-\frac{C_o}{H}t}\right) \tag{25}$$

Equation (25) gives the altitude climbed in any time t. In the form

$$t_c = \frac{H}{C_o} \log_e \left(\frac{H}{H-y} \right) \tag{25a}$$

it gives the time required to climb to any altitude y.

RANGE.

The common formula for range, usually credited to Breguet, is easily derived. The velocity varies as the square root of the weight W

$$V = K_1 \sqrt{W} \tag{26}$$

The thrust horsepower is

$$THP = \frac{WV}{K_2 \left(\frac{L}{D} \right)} = \frac{K_1 W^{3/2}}{K_2 \left(\frac{L}{D} \right)} \tag{27}$$

now

$$\frac{dW}{dt} = \frac{THP}{\eta} \cdot c$$

where c is the specific fuel consumption and η the propeller efficiency. Therefore

$$dt = \frac{dW \cdot \eta}{THP \cdot c} = \frac{K_2 \cdot \eta}{c} \frac{L}{D} \cdot \frac{dW}{K_1 W^{3/2}} \tag{28}$$

The range is given by

$$R = \int V\, dt = \int_{W_1}^{W_2} \left(K_1 \sqrt{W} \cdot K_2 \cdot \frac{\eta}{c} \frac{L}{D} \frac{dW}{K_1 W^{3/2}} \right) = \frac{\eta}{c} \left(\frac{L}{D} \right) K_2 \int_{W_1}^{W_2} \frac{dW}{W}$$

$$R = K_2 \cdot \left(\frac{L}{D} \right) \frac{\eta}{c} \log_e \frac{W_1}{W_2} \tag{29}$$

When W is in lb., V in M. P. H., R will be in miles and K_2 will have the value 375, so that

$$R = 375 \left(\frac{L}{D} \right) \left(\frac{\eta}{c} \right) \log_e \frac{W_1}{W_2} \tag{30}$$

or

$$R = 863 \left(\frac{L}{D} \right) \frac{\eta}{c} \log_{10} \frac{W_1}{W_2} \tag{30a}$$

In this equation W_1 is the weight fully loaded and W_2 the weight W_1 less fuel.

It will be noted that this formula does not contain a density term. The range is therefore independent of the air density except as the term $\left(\frac{\eta}{c} \right)$ is affected.

ENDURANCE.

The endurance at any speed and for a given fuel load is obtained by direct integration of equation (28)

$$dt = \frac{K_2}{K_1} \cdot \frac{\eta}{c} \cdot \left(\frac{L}{D} \right) \frac{dW}{W^{3/2}} \tag{28}$$

$$t = \frac{K_2}{K_1} \cdot \frac{\eta}{c} \cdot \frac{L}{D} \int_{W_1}^{W_2} \frac{dW}{W^{3/2}} = \frac{K_2}{K_1} \cdot \frac{\eta}{c} \cdot \frac{L}{D} \cdot 2 \left\{ \frac{1}{\sqrt{W_2}} - \frac{1}{\sqrt{W_1}} \right\} \tag{31}$$

When W is in lb. and c in lb./BHP/hr., T will be given in hours if $K=375$ and $K_1=\sqrt{\frac{2}{C_L\rho S}}$. That is, in hours

$$T=\frac{750}{\sqrt{\frac{2}{C_L\rho S}}}\cdot\frac{\eta}{c}\cdot\frac{L}{D}\left(\frac{1}{\sqrt{W_2}}-\frac{1}{\sqrt{W_1}}\right)=750\frac{V}{\sqrt{W}}\cdot\frac{\eta}{c}\,\frac{L}{D}\left(\frac{1}{\sqrt{W_2}}-\frac{1}{\sqrt{W_1}}\right) \tag{32}$$

Equation (32) gives the time required at a fixed angle of attack (and corresponding $\frac{L}{D}$ and C_L) to consume (W_1-W_2) lb. of fuel. Note that this time depends directly on the square root of the density if the effect of density on the term $\frac{\eta}{c}$ be ignored. However, the variation of η with ρ must be calculated for each case if great accuracy is required. A rough approximation based on test data is

$$\eta=\eta_o\left(1+\frac{y}{2000}\right) \tag{33}$$

That is, the propeller efficiency increases about 1 per cent for each 2,000 feet of altitude. The variation of the specific fuel consumption with altitude may be obtained from National Advisory Committee for Aeronautics Technical Reports Nos. 46, 102, 103, 134, or 135. The average relative values of c are as follows:

y ft.	c – relative
0	1.00
5000	1.03
10000	1.11
15000	1.19
20000	1.46
25000	2.26

COLLECTED FORMULAE.

Stalling speed $V_S=19.8\sqrt{\frac{\frac{W}{S}}{C_{LM}}}$ M. P. H.

Climbing speed $V_c=\frac{(2V_S+V_M)}{3}$ M. P. H.

Speed range ratio $\frac{V_M}{V_S}=\frac{20.3\,\eta^{1/3}}{\sqrt[3]{V_S\cdot\left(\frac{W}{HP}\right)}}=\frac{10.2\,\eta^{1/3}\left(\frac{L}{D}\right)_M^{\frac{1}{3}}}{\sqrt[3]{V_S\cdot\frac{W}{HP}}}$

Initial climb $C_o=33000\left(\frac{K_2\,\eta_m}{\left(\frac{W}{HP}\right)}-\frac{(2V_S+V_M)}{1125\left(\frac{L}{D}\right)}\right)$ ft./min.

Absolute ceiling $H_a=f\left(\frac{HP_{ao}}{HP_{ro}}\right)$

$$\frac{HP_{ao}}{HP_{ro}}=\frac{61.7\left(\frac{L}{D}\right)}{\left(\frac{1}{\eta_m}\cdot V_S\cdot\frac{W}{HP}\right)^{080}}$$

Service ceiling $H_S=H_a\,\frac{(C_o-100)}{C_o}$ ft.

Climb in given time $y=H_a\left(1-e^{-\frac{C_o}{H_s}t}\right)$ft.

Time of climb $t_c=\frac{H_a}{C_o}\cdot\log_e\left(1-\frac{y}{H}\right)$minutes.

Range $R=863\left(\frac{L}{D}\right)\left(\frac{\eta}{c}\right)\log_{10}\left(\frac{W_1}{W_2}\right)$miles.

Endurance $T=750\frac{V}{\sqrt{W}}\left(\frac{\eta}{c}\right)\left(\frac{L}{D}\right)\left(\frac{1}{\sqrt{W_2}}-\frac{1}{\sqrt{W_1}}\right)$hrs.

In these formulae the following units are to be used with the constants given:

V_S, V_M, V_c	M. P. H.
C_o	ft./min.
H_a, H_S, y	feet.
Time of climb t_c	minutes.
Range R	miles.
Specific fuel consumption c	lb./BHP/hr.
Weight W	lb.
Endurance T	hours.

TABLE I.

SHOWING RANGE OF LIFT COEFFICIENT FOR STANDARD AIRFOILS FOR WHICH $\left(\frac{L}{D}\right)=(C_L)\times$CONSTANT

Airfoil.	Max. C_L C_{LM}	$\frac{d\left(\frac{L}{D}\right)}{d\alpha}$ =const. from $C_L=0$ to C_{L_1}	$\sqrt{\frac{C_{LM}}{C_{L_1}}}$	Reference.
USA-1	1.14	0.46	1.57	N. A. C. A. Report No. 93.
USA-4	1.44	.65	1.49	N. A. C. A. Report No. 93.
USA-16	0.99	.35	1.68	N. A. C. A. Report No. 93.
RAF-6	1.22	.50	1.56	N. A. C. A. Report No. 93.
RAF-14	1.08	.48	1.50	N. A. C. A. Report No. 93.
RAF-15	1.03	.40	1.60	N. A. C. A. Report No. 93.
RAF-19	1.69	.96	1.33	N. A. C. A. Report No. 93.
Albatros	1.35	.62	1.48	N. A. C. A. Report No. 93.
USA-27	1.40	.55	1.59	N. A. C. A.
Göttingen 256	1.21	.60	1.42	N. A. C. A.
Göttingen 387	1.36	.45	1.73	N. A. C. A.
Average			1.54	

NOTE.—This data shows $\frac{L}{D}=C_L\times$constant for all lift coefficients less than $\left(\frac{1}{1.54}\right)^2 C_{L_{max}}=0.42\ C_{L_{max}}$.

TABLE II.

WIND TUNNEL TEST DATA ON AIRPLANE MODEL CORRECTED TO 300 SQ. FT. WING AREA.

α	Lift at 40 M. P. H.	Drag at 40 M. P. H.	$\frac{L}{D}$	$\frac{V}{V_S}$
−1	119.2	58.6	2.034	3.315
0	203.4	58.2	3.495	2.540
1	288.3	58.7	4.910	2.133
2	369.9	60.7	6.094	1.882
3	451.1	64.0	7.048	1.705
4	530.4	68.7	7.721	1.571
6	688.9	81.2	8.484	1.377
8	844.1	97.7	8.640	1.244
10	997.8	118.0	8.456	1.144
12	1,143.3	143.0	7.995	1.070
14	1,270.5	177.3	7.166	1.015
16	1,325.0	242.7	5.395	1.000

Minimum speed $V_o=40\sqrt{\frac{W}{1325.0}}=\sqrt{1.2\,W}$.

TABLE II-A.

FAIRED VALUES OF $\frac{L}{D}$ VERSUS $\frac{V}{V_s}$

TAKEN FROM FIG. 1.

$\frac{V}{V_s}$	$\frac{L}{D}$
1.00	5.40
1.10	8.20
1.20	8.60
1.30	8.62
1.50	8.02
1.70	7.10
2.00	5.60
2.30	4.25
2.60	3.35
2.90	2.70
3.00	2.53

TABLE III.

POWER REQUIRED AND POWER AVAILABLE FOR $\frac{W}{S}$=4 LB./FT.^{2}, $\frac{W}{HP}$=6 LB./BHP, METHOD OF CALCULATION.

$\frac{V}{V_s}$	$\frac{L}{D}$	V M. P. H.	D $\frac{W}{\frac{L}{D}}$	HPr $\frac{DV}{375}$	N R. P. M.	$\frac{V}{ND}$	$\frac{\frac{V}{ND}}{\left(\frac{V}{ND}\right)_o}$	$\frac{\eta}{\eta_o}$	η	BHP	HP_a
1.00	5.40	38.0	221	22.4	1,600	0.256	0.371	0.590	0.460	177.8	82
1.10	8.20	41.8	146	16.3	1,610	.281	.407	.630	.492	179.0	88
1.20	8.60	45.6	140	17.0	1,620	.304	.442	.674	.526	180.0	95
1.30	8.62	49.4	139	18.3	1,630	.327	.475	.710	.554	181.0	100
1.50	8.02	57.0	150	22.7	1,650	.373	.541	.775	.605	183.3	111
1.70	7.10	64.6	169	29.1	1,670	.418	.607	.830	.648	185.6	120
2.00	5.60	76.0	214	43.4	1,700	.483	.700	.900	.702	189.0	132
2.30	4.25	87.4	282	65.7	1,730	.546	.790	.947	.738	192.2	142
2.60	3.35	98.8	358	94.3	1,760	.606	.880	.980	.765	195.6	150
2.90	2.70	110.2	444	130.5	1,790	.666	.966	.997	.778	199.0	155
3.00	2.53	114.0	475	144.5	1,800	.684	.992	1.000	.780	200.0	156

$\frac{\eta}{\eta_o}$ is from N. A. C. A. Report No. 168.

TABLE IV.

HP_r FOR $\frac{W}{S}$=4 LB./FT.^{2} AND HP_a FOR VARIOUS POWER LOADINGS.

$\frac{V}{V_o}$	V	D	HP_r	HP_a for $\frac{W}{HP}$ as indicated.					
				6	8	11	16	20	24
1.00	38.0	221	22.4	82	65.1	50.4	40.6	31.2	27.2
1.10	41.8	146	16.3	88	70.2	54.0	43.2	33.2	29.8
1.20	45.6	140	17.0	95	75.3	57.7	45.7	35.2	30.3
1.30	49.4	139	18.3	100	79.0	61.3	47.9	36.7	32.0
1.50	57.0	149	22.7	111	87.6	66.5	51.9	39.7	34.0
1.70	64.6	169	29.1	120	94.6	71.6	55.0	41.9	35.6
2.00	76.0	214	43.4	132	104.0	77.7	57.5	44.2	36.9
2.30	87.4	282	65.7	142	109.6	81.5	59.7		
2.60	98.8	358	94.3	150	114.3	84.0			
2.90	110.2	444	130.5	155	116.5				
3.00	114.0	475	144.5	156					

TABLE V.

HP_r FOR $\frac{W}{S}$=6 AND HP_a FOR VARIOUS POWER LOADINGS.

$\frac{V}{V_o}$	V	D	HP_r	HP_a for $\frac{W}{HP}$ as indicated.					
				6	8	11	16	20	24
1.00	46.5	333	41.3	128.6	104.3	81.1	59.8	49.6	43.0
1.10	51.1	219	30.0	138.1	112.2	86.3	63.5	53.0	45.5
1.20	55.8	209	31.1	147.2	119.5	92.3	67.5	55.7	47.8
1.30	60.4	208	33.6	156.6	126.6	97.1	70.8	58.2	49.8
1.50	69.7	224	41.7	172.5	138.2	105.6	76.2	62.5	53.1
1.70	79.0	253	53.4	187.5	149.0	112.4	81.0	65.8	55.5
2.00	93.0	321	79.6	205.0	161.5	121.0	85.2	68.6	
2.30	107.0	423	120.8	221.0	171.0	126.3	86.0		
2.60	120.9	537	173.3	231.0	176.0	127.5			
2.90	134.9	665	240.0	237.0					
3.00	139.5	711	265.0						

TABLE VI.

HP_r FOR $\frac{W}{S}=8$ AND HP_a FOR VARIOUS POWER LOADINGS.

$\frac{V}{V_0}$	V	D	HP_r	HP_a for $\frac{W}{HP}$ as indicated.					
				6	8	11	16	20	24
1.00	53.6	444	63.5	180.5	144.2	112.5	83.2	69.5	59.5
1.10	59.0	293	46.0	194.5	153.5	119.7	88.6	73.5	62.8
1.20	64.3	279	47.8	207.0	163.5	127.0	93.5	77.2	65.8
1.30	69.7	278	51.8	219.0	173.5	134.0	97.5	80.7	68.5
1.50	80.4	299	64.1	241.0	189.0	145.3	105.1	86.2	72.8
1.70	91.1	338	82.1	261.0	203.0	154.0	111.0	91.1	75.5
2.00	107.2	428	122.7	285.0	220.0	165.5	116.3	92.7	
2.30	123.3	565	186.0	303.0	232.0	171.0			
2.60	139.4	717	266.0	316.0	238.0				
2.90	155.4	888	369.0	321.0					
3.00	160.8	948	407.0						

TABLE VII.

HP_r FOR $\frac{W}{S}=10$ AND HP_a FOR VARIOUS POWER LOADINGS.

$\frac{V}{V_0}$	V	D	HP_r	HP_a for $\frac{W}{HP}$ as indicated.					
				6	8	11	16	20	24
1.00	60.0	556	89.0	232	178	145.0	108.3	89.0	76.5
1.10	66.0	366	64.5	250	193	154.0	113.3	94.0	81.0
1.20	72.0	349	67.0	267	207	164.0	119.8	98.5	84.7
1.30	78.0	348	72.3	282	219	172.0	125.8	102.7	87.8
1.50	90.0	374	90.0	310	245	186.5	134.8	109.5	92.8
1.70	102.0	422	115.0	335	262	198.0	141.5	114.5	96.5
2.00	120.0	536	171.5	363	282	211.0	147.4		
2.30	138.0	706	260.0	385	295	216.0			
2.60	156.0	896	373.0	400	302				
2.90	174.0	1,110	515.0	403					

TABLE VIII.

HP_r FOR $\frac{W}{S}=14$ AND HP_a FOR VARIOUS POWER LOADINGS.

$\frac{V}{V_0}$	V	D	HP_r	HP_a for $\frac{W}{HP}$ as indicated.					
				6	8	11	16	20	24
1.00	71.0	778	147.3	342	272	212	155.5	130.6	111.5
1.10	78.1	512	106.5	366	292	226	165.0	138.0	117.7
1.20	85.2	488	111.0	389	309	239	173.5	144.5	123.0
1.30	92.3	487	120.0	411	326	250	181.0	150.0	128.0
1.50	106.5	523	149.0	450	354	271	194.5	159.5	134.7
1.70	120.7	592	190.0	484	377	286	203.0	164.6	
2.00	142.0	750	284.0	522	404	303	206.0		
2.30	163.3	988	431.0	553	422	305			
2.60	184.6	1,254	618.0	568					
2.90	205.9	1,660	855.0						

TABLE IX.

PERFORMANCE FOR $\frac{W}{S}=4$ AND VARIOUS $\frac{W}{HP}$ WITH DETERMINATION OF K_1.

Wing loading $\frac{W}{S}$, lb./ft.²	4	4	4	4	4	4
Power loading $\frac{W}{HP}$, lb./BHP	6	8	11	16	20	24
Weight	1,200	1,200	1,200	1,200	1,200	1,200
BHP	200	150	109	75	60	50
Minimum speed V_S	38	38	38	38	38	38
Maximum speed V_M	117.5	105.8	94.8	82.9	76.5	71
Speed range $\frac{V_M}{V_S}$	3.09	2.78	2.50	2.18	2.01	1.87
[illegible]	91.5	65.2	44.0	26.5	18.1	13.6
[illegible]	2,510	1,795	1,210	730	495	375
[illegible]	5.56	4.44	3.40	2.54	2.07	1.80
[illegible]	32,500	28,600	24,300	19,100	15,900	12,500
[illegible]	31,200	27,000	22,300	16,500	12,200	9,200
$V_S\times\frac{''}{HP}$	228	304	418	608	760	912
$\left(V_S\times\frac{W}{HP}\right)^{1/3}$	6.109	6.724	7.447	8.472	9.126	9.698
$K_1\times\sqrt[3]{\eta}$	18.87	18.70	18.70	18.50	18.33	18.22
η	0.78	0.775	0.765	0.740	0.736	0.730
$\sqrt[3]{\eta}$	.920	.918	.914	.905	.903	.900
K_1	20.50	20.35	20.45	20.45	20.30	20.25

TABLE X.

PERFORMANCE FOR $\frac{W}{S}=6$ AND VARIOUS $\frac{W}{HP}$ WITH DETERMINATION OF K_1.

Wing loading $\frac{W}{S}$, lb./ft.²	6	6	6	6	6	6
Power loading $\frac{W}{HP}$, lb./BHP	6	8	11	16	20	24
Weight	1,800	1,800	1,800	1,800	1,800	1,800
BHP	300	225	163.6	112.5	90	75
Minimum speed V_S	46.5	46.5	46.5	46.5	46.5	46.5
Maximum speed V_M	134.0	121.3	108.6	95.6	88.0	81.0
Speed range $\frac{V_M}{V_S}$	2.88	2.61	2.33	2.06	1.89	1.74
[illegible]	134	96.5	64	37	24.5	17
[illegible]	2,450	1,770	1,175	680	450	310
[illegible]	4.75	3.91	2.96	2.17	1.80	1.55
[illegible]	29,700	26,500	21,800	16,200	12,500	9,500
[illegible]	28,500	25,000	19,900	13,800	9,700	6,400
$V_S\times\frac{''}{HP}$	279	372	511.5	744	930	1,116
$\left(V_S\times\frac{W}{HP}\right)^{1/3}$	6.534	7.192	7.997	9.061	9.761	10.373
$K_1\times\sqrt[3]{\eta}$	18.80	18.75	18.65	18.65	18.44	18.06
η	0.790	0.783	0.775	0.765	0.756	0.742
$\sqrt[3]{\eta}$	.923	.921	.918	.914	.909	.905
K_1	20.35	20.35	20.30	20.40	20.30	19.95

TABLE XI.

PERFORMANCE FOR $\frac{W}{S}=8$ AND VARIOUS $\frac{W}{HP}$ WITH DETERMINATION OF K_1.

Wing loading $\frac{W}{S}$, lb./ft.²	8	8	8	8	8	8
Power loading $\frac{W}{HP}$, lb./BHP	6	8	11	16	20	24
Weight	2,400	2,400	2,400	2,400	2,400	2,400
BHP	400	300	218	150	120	100
Minimum speed V_S	53.6	53.6	53.6	53.6	53.6	53.6
Maximum speed V_M	148.2	134.0	119.8	105.0	96.5	87.3
Speed range $\frac{V_M}{V_S}$	2.77	2.50	2.235	1.96	1.80	1.63
[illegible]	178.6	124	82.3	45.7	30.5	18.6
[illegible]	2,450	1,705	1,130	630	420	255
[illegible]	4.38	3.47	2.70	1.97	1.65	1.39
[illegible]	28,400	24,600	20,200	14,200	10,700	7,300
[illegible]	27,200	23,200	18,400	12,000	8,200	4,400
$V_S\times\left(\frac{''}{HP}\right)$	321.6	428.8	589.6	857.6	1,072	1,286.4
$\left(V_S\times\frac{W}{HP}\right)^{1/3}$	6.851	7.541	8.385	9.501	10.234	10.875
$K_1\times\sqrt[3]{\eta}$	18.96	18.85	18.73	18.62	17.42	17.75
η	0.800	0.794	0.784	0.775	0.762	0.747
$\sqrt[3]{\eta}$	.928	.925	.921	.918	.913	.907
K_1	20.45	20.40	20.35	20.30	20.15	19.56

TABLE XII.

PERFORMANCE FOR $\frac{W}{S}=10$ WITH VARIOUS $\frac{W}{HP}$ WITH DETERMINATION OF K_1.

Wing loading $\frac{W}{S}$, lb./ft.²	10	10	10	10	10	10
Power loading $\frac{W}{HP}$, lb./BHP	6	8	11	16	20	24
Weight	3,000	3,000	3,000	3,000	3,000	3,000
BHP	500	375	273	187.5	150	125
Minimum speed V_S	60	60	60	60	60	60
Maximum speed V_M	160.1	145.8	129.3	113.2	102.1	92.3
Speed range $\frac{V_M}{V_S}$	2.67	2.43	2.155	1.886	1.700	1.538
[illegible]	219.5	152.5	98.2	51.3	31.0	17.0
[illegible]	2,415	1,680	1,080	565	340	187
[illegible]	3.96	3.07	2.42	1.76	1.455	1.256
[illegible]	26,700	22,500	18,200	12,000	8,200	5,100
[illegible]	25,600	21,200	16,500	9,900	5,800	2,370
$V_S \times \left(\frac{W}{HP}\right)$	360	480	660	960	1,200	1,440
$\left(V_S \times \frac{W}{HP}\right)^{1/3}$	7.114	7.830	8.707	9.865	10.626	11.293
$K_1 \times \sqrt[3]{\eta}$	18.95	19.00	18.78	18.58	18.08	17.38
η	0.805	0.798	0.790	0.780	0.767	0.754
$\sqrt[3]{\eta}$	.930	.927	.923	.920	.915	.910
K_1	20.35	20.45	20.30	20.20	19.75	19.12

TABLE XIII.

PERFORMANCE FOR $\frac{W}{S}=14$ WITH VARIOUS $\frac{W}{HP}$ WITH DETERMINATION OF K_1.

Wing loading $\frac{W}{S}$, lb./ft.²	14	14	14	14	14	14
Power loading $\frac{W}{HP}$, lb./BHP	6	8	11	16	20	24
Weight	4,200	4,200	4,200	4,200	4,200	4,200
BHP	700	525	382	262.5	210	175
Minimum speed V_S	71	71	71	71	71	71
Maximum speed V_M	179.1	162.8	145.7	124.7	111.4	98.5
Speed range V_M/V_S	2.52	2.29	2.05	1.76	1.57	1.388
Maximum excess HP	299	205	128	62	33	14
Initial climb, ft./min.	2,350	1,610	1,005	485	260	110
Maximum (HP_{so}/HP_{ro})	3.54	2.81	2.15	1.575	1.306	1.155
[illegible]	24,900	20,800	16,000	9,800	6,000	3,200
[illegible]	23,800	19,500	14,400	7,800	3,700	290
$V_S \times \left(\frac{W}{HP}\right)$	426	568	781	1,136	1,420	1,704
$\left(V_S \times \frac{W}{HP}\right)^{1/3}$	7.524	8.282	9.209	10.434	11.241	11.944
$K_1 \times \sqrt[3]{\eta}$	18.95	18.95	18.85	18.42	17.67	16.58
η	0.810	0.805	0.796	0.788	0.772	0.755
$\sqrt[3]{\eta}$	.932	.930	.927	.923	.917	.909
K_1	20.35	20.35	20.30	19.95	19.30	18.25

TABLE XIV.

VALUE OF K_1 FROM OBSERVED PERFORMANCE $\frac{V_M}{V_S}=\frac{K_1\eta^{1/3}}{\left(V_S \cdot \frac{W}{HP}\right)^{1/3}}$

Airplane.	$\frac{W}{HP}$	V_M	V_S	$\frac{V_M}{V_S}$	η_m	$\eta_m^{1/3}$	$V_S \times \frac{W}{HP}$	$F^{1/3}$	K
MB-3	5.8	152	55	2.77	0.81	0.933	319	6.832	20.30
Le Pere	9.0	136	58	2.34	.80	.927	522	8.050	20.30
Spad 13	9.2	132	59	2.24	.76	.911	542	8.154	20.05
TS-1	10.1	118	50	2.36	.79	.924	505	7.963	20.30
TR-1	8.96	130	55	2.36	.79	.924	493	7.900	20.10
NW	5.10	200	74	2.70	.83	.940	377	7.224	20.75
18-T-1	7.60	160	66	2.42	.81	.933	502	7.948	20.60
HA	10.30	127	59	2.15	.76	.912	608	8.472	20.00
DH4	10.70	124	60	2.07	.77	.916	642	8.627	19.50
VE-7	10.50	120	52	2.31	.78	.920	546	8.173	20.50
SE-5	11.40	122	57	2.14	.77	.916	650	8.660	20.25
JN-4H	14.30	93	44	2.11	.74	.903	630	8.573	20.05
Messenger	13.50	97	45	2.16	.78	.920	608	8.472	19.85
DT-2	16.10	100	52	1.92	.74	.903	838	9.428	20.10
F-5L	18.10	87	53	1.64	.68	.880	959	9.860	18.35
N-9H	18.32	78	42	1.86	.72	.896	770	9.166	19.05

TABLE XV.

SHOWING THE RELATION BETWEEN CLIMBING SPEED V_c, STALLING SPEED V_S, AND MAXIMUM SPEED V_M, BASED ON DATA IN TABLES IX-XIII.

V_S	$\frac{W}{HP}=$	6	8	11	16	20	24
38	V_M	117.5	105.8	94.8	82.9	76.5	71
	V_c	67	63	59	52	50	48
	(V_M-V_S)	79	67.8	56.8	44.9	38.5	33
	(V_c-V_S)	29	25	21	14	12	10
	$(V_c-V_S)\div(V_M-V_S)$	0.367	0.367	0.368	0.312	0.312	0.303
46.5	V_M	134	121.3	108.6	95.6	88	81
	V_c	80	73	67	62	60	57
	(V_M-V_S)	87.5	74.8	62.1	49.1	41.5	34.5
	(V_c-V_S)	33.5	26.5	20.5	15.5	13.5	10.5
	$(V_c-V_S)\div(V_M-V_S)$	0.383	0.354	0.330	0.315	0.325	0.305
53.6	V_M	148.2	134.0	119.8	105	96.5	87.3
	V_c	88	82.5	74	68	65	63
	(V_M-V_S)	94.6	80.2	66.0	51.4	42.9	33.7
	(V_c-V_S)	34.4	28.9	20.4	14.4	11.4	9.4
	$(V_c-V_S)\div(V_M-V_S)$	0.363	0.360	0.319	0.280	0.266	0.280
60	V_M	160.1	145.8	129.3	113.2	102.1	92.3
	V_c	96	89	81	75	73	70
	(V_M-V_S)	100.1	85.8	69.3	53.2	42.1	32.3
	(V_c-V_S)	36	29	21	15	13	10
	$(V_c-V_S)\div(V_M-V_S)$	0.360	0.338	0.303	0.282	0.309	0.310
71	V_M	179.1	162.8	145.7	124.7	111.4	98.5
	V_c	107	99	92	86	83	83
	(V_M-V_S)	108.1	91.8	74.7	54.7	40.4	27.5
	(V_c-V_S)	36	28	21	15	12	11
	$(V_c-V_S)\div(V_M-V_S)$	0.333	0.305	0.282	0.278	0.297	0.292

TABLE XVI.

DETERMINATION OF K_2 IN THE EQUATION FOR INITIAL RATE OF CLIMB $C_o=33000\left[\frac{K_2\,\eta_m}{\left(\frac{W}{HP}\right)}-\frac{(2V_S+V_M)}{1125\left(\frac{L}{D}\right)}\right]$

V_S	$W/HP=$	6	8	11	16	20	24
38	V_M/V_S	3.09	2.78	2.50	2.18	2.01	1.87
		67	63	59	52	50	48
		2,510	1,795	1,210	730	495	375
	$V_c\div3{,}230$	0.02075	0.01950	0.01828	0.01610	0.01545	0.01484
	$C_o\div33{,}000$	.07610	.05440	.03670	.02210	.01500	.01137
	$K_2\eta\div(W/HP)$	.09685	.07390	.05498	.03820	.03045	.02621
	η	.780	.775	.756	.740	.736	.730
	K_2	.745	.763	.790	.827	.828	.803
46.5	V_M/V_S	2.88	2.61	2.333	2.056	1.89	1.742
	Speed for climb, V_c	80	73	67	62	60	57
	Actual initial climb, C_o	2,450	1,770	1,175	680	450	310
	$V_c\div3{,}230$	0.02470	0.02260	0.02070	0.01920	0.01855	0.01765
	$C_o\div33{,}000$	.07420	.05360	.03560	.02030	.01365	.00940
	$K_2\eta\div(W/HP)$	.09890	.07620	.05630	.03980	.03220	.02705
	η	.790	.783	.775	.765	.756	.742
	K_2	.752	.777	.798	.834	.853	.876
53.6	V_M/V_S	2.77	2.50	2.235	1.96	1.80	1.63
	Speed for climb, V_c	88	82.5	74	68	65	63
	Actual initial climb, C_o	2,450	1,705	1,130	630	420	255
		0.02720	0.02550	0.02290	0.02105	0.02010	0.01950
		.07420	.05170	.03430	.01910	.01273	.00773
		.10140	.07720	.05720	.04015	.03283	.02723
	η	.800	.794	.784	.775	.762	.747
	K_2	.761	.779	.803	.831	.862	.876
60	V_M/V_S	2.67	2.43	2.155	1.886	1.70	1.538
	Speed for climb, V_c	96	89	81	75	73	70
	Actual initial climb, C_o	2,415	1,680	1,080	565	340	187
	$V_c\div3{,}230$	0.02970	0.02750	0.02505	0.02320	0.02260	0.02165
	$C_o\div33{,}000$	.07320	.05090	.03275	.01712	.01030	.00566
	$K_2\eta\div(W/HP)$	.10290	.07840	.05780	.04032	.03290	.02731
	η	.805	.798	.790	.780	.767	.754
	K_2	.767	.784	.805	.829	.858	.870
71	V_M/V_S	2.52	2.29	2.05	1.76	1.57	1.388
	Speed for climb, V_c	107	99	92	86	83	82
	Actual initial climb, C_o	2,350	1,610	1,005	485	260	110
	$V_c\div3{,}230$	0.03320	0.03030	0.02845	0.02660	0.02570	0.02535
	$C_o\div33{,}000$	.07120	.04875	.03050	.01470	.00788	.00333
	$K_2\eta\div(W/HP)$	.10440	.07935	.05895	.04130	.03358	.02868
	η	.810	.805	.796	.788	.772	.755
	K_2	.775	.788	.815	.840	.872	.913

TABLE XVII.

COMPARISON OF OBSERVED RATE OF CLIMB WITH THAT CALCULATED BY FORMULA $C_o=33000\left[\frac{K_1\,\eta_m}{\left(\frac{W}{HP}\right)}-\frac{(2V_S+V_M)}{1125\left(\frac{L}{D}\right)}\right]$

Airplane.	$\frac{W}{HP}$	V_M	V_S	V_C	$\frac{V_M}{V_S}$	K_1	η_m	$\frac{L}{D}$	$\frac{K\eta_m}{\frac{W}{HP}}$	$\frac{V_c}{375\frac{L}{D}}$	F	Climb. Calculated $\frac{33,000}{F}$	Climb. Actual.
US×BIA	9.98	133.0	54	80	2.46	0.785	0.790	8	0.0622	0.0267	0.0355	1,170	1,300
MB 3	7.00	152.0	58	89	2.62	.770	.760	8	.0835	.0296	.0531	1,750	1,930
M 80	8.80	143.5	63	90	2.28	.800	.780	8	.0708	.0300	.0408	1,350	1,510
"D"	8.10	147.0	55	86	2.67	.765	.750	8	.0707	.0287	.0420	1,380	1,460
S 6	17.60	97.0	45	62	2.15	.810	.805	8	.0371	.0207	.0164	540	690
Roland D-VI-B	9.94	114.0	56	75	2.04	.825	.780	8	.0648	.0250	.0398	1,310	1,230
JL-6	14.80	111.2	52	72	2.14	.810	.760	8	.0417	.0240	.0177	580	580
[illegible]	13.50	96.7	44	61	2.20	.810	.780	8	.0467	.0204	.0263	860	700
[illegible]	17.60	94.3	51	65	1.85	.845	.800	8	.0385	.0217	.0168	560	615
[illegible]	17.60	85.3	42	56	2.03	.825	.780	8	.0365	.0187	.0178	590	700
Spad 13	9.16	131.5	60	84	2.19	.810	.780	8	.0689	.0280	.0409	1,350	1,200
DH-4	10.20	123.7	62	83	2.00	.830	.790	8	.0644	.0277	.0367	1,210	1,000
Fokker D-VIII	9.00	115.0	57	76	2.02	.830	.790	8	.0728	.0254	.0474	1,560	1,500
VE-7	11.60	116.5	52	74	2.24	.805	.790	8	.0549	.0247	.0302	1,000	900
SE-5	11.40	121.6	54	76	2.25	.805	.790	8	.0559	.0254	.0305	1,010	1,010

TABLE XVIII.

DETERMINATION OF K_3 IN THE EQUATION $\frac{HP_{ac}}{HP_{ro}}=\frac{K_3\cdot\eta_m\cdot\frac{L}{D}}{V_S\cdot\frac{W}{HP}}$

V_S	$W/HP=$	6	8	11	16	20	24
38	$V_S\cdot W/HP$	228	304	418	608	760	912
	η_m	0.780	0.775	0.765	0.740	0.736	0.730
	$(1/\eta_m)\cdot V_S\cdot(W/HP)$	292	392	546	822	1,033	1,248
	$(L/D)\div(1/\eta_m)\cdot V_S\cdot(W/HP)=A$	0.02940	0.02190	0.01570	0.01047	0.00833	0.00688
	HP_{ac}/HP_{ro}	5.56	4.44	3.40	2.54	2.07	1.80
	$K_3=(HP_{ac}/HP_{ro})\div A$	189	203	216	243	248	262
46.5	$V_S\cdot W/HP$	279	372	511.5	744	930	1,116
	η_m	0.790	0.783	0.775	0.765	0.756	0.742
	$(1/\eta_m)\cdot V_S\cdot(W/HP)$	353	475	660	972	1,230	1,506
	$(L/D)\div(1/\eta_m)\cdot V_S\cdot(W/HP)=A$	0.02430	0.01812	0.01302	0.00884	0.00698	0.00573
	HP_{ac}/HP_{ro}	4.75	3.91	2.96	2.17	1.80	1.55
	$K_3=(HP_{ac}/HP_{ro})\div A$	195	215	227	246	258	271
53.6	$V_S\cdot W/HP$	321.6	428.8	589.6	857.6	1,072	1,286.4
	η_m	0.800	0.794	0.784	0.775	0.762	0.747
	$(1/\eta_m)\cdot V_S\cdot(W/HP)$	402	540	752	1,106	1,408	1,720
	$(L/D)\div(1/\eta_m)\cdot V_S\cdot(W/HP)=A$	0.02135	0.01593	0.01144	0.00777	0.00611	0.00500
	HP_{ac}/HP_{ro}	4.38	3.47	2.70	1.97	1.65	1.39
	$K_3=(HP_{ac}/HP_{ro})\div A$	205	218	235	253	270	278
60	$V_S\cdot W/HP$	360	480	660	960	1,200	1,440
	η_m	0.805	0.798	0.790	0.780	0.767	0.754
	$(1/\eta_m)\cdot V_S\cdot(W/HP)$	447	608	835	1,230	1,564	1,909
	$(L/D)\div(1/\eta_m)\cdot V_S\cdot(W/HP)=A$	0.01923	0.01430	0.01028	0.00698	0.00550	0.00451
	HP_{ac}/HP_{ro}	3.96	3.07	2.42	1.76	1.455	1.256
	$K_3=(HP_{ac}/HP_{ro})\div A$	206	215	235	252	265	279
71	$V_S\cdot W/HP$	426	568	781	1,136	1,420	1,704
	η_m	0.810	0.805	0.796	0.788	0.772	0.755
	$(1/\eta_m)\cdot V_S\cdot(W/HP)$	526	706	981	1,441	1,838	2,255
	$(L/D)\div(1/\eta_m)\cdot V_S\cdot(W/HP)=A$	0.01634	0.01218	0.00878	0.00597	0.00468	0.00380
	HP_{ac}/HP_{ro}	3.54	2.81	2.15	1.575	1.306	1.155
	$K_3=(HP_{ac}/HP_{ro})\div A$	216	231	245	264	279	304

TABLE XIX.

DETERMINATION OF K_4 IN THE ABSOLUTE CEILING FORMULA $\frac{HP_{ao}}{HP_{ro}}=\frac{K_4\frac{L}{D}}{\left(\frac{1}{\eta_m}\cdot V_s\cdot\frac{W}{HP}\right)^{0}}$

V_s	$W/HP=$	6	8	11	16	20	24
38	$(1/\eta_m)\cdot V_s\cdot(W/HP)=B$	292	392	546	822	1,033	1,248
	$B^{0.80}$	93.8	118.6	154.7	214.2	258	300
	HP_{ao}/HP_{ro}	5.56	4.44	3.40	2.54	2.07	1.80
	$K_4=(HP_{ao}/HP_{ro})\,B^{0.80}\div\frac{L}{D}$	60.6	61.2	61.2	63.2	62.0	62.7
46.5	$(1/\eta_m)\cdot V_s\cdot(W/HP)=B$	353	475	660	972	1,230	1,506
	$B^{0.80}$	109.2	138.6	180.0	245.0	296.5	349.0
	HP_{ao}/HP_{ro}	4.75	3.91	2.96	2.17	1.80	1.55
	$K_4=(HP_{ao}/HP_{ro})\,B^{0.80}\div\frac{L}{D}$	60.2	62.9	61.9	61.9	61.9	62.7
53.6	$(1/\eta_m)\cdot V_s\cdot(W/HP)=B$	402	540	752	1,106	1,408	1,720
	$B^{0.80}$	121.3	153.4	200.3	272	330	387
	HP_{ao}/HP_{ro}	4.38	3.47	2.70	1.97	1.65	1.39
	$K_4=(HP_{ao}/HP_{ro})\,B^{0.50}\div\frac{L}{D}$	61.7	61.8	62.8	62.2	63.1	62.5
60	$(1/\eta_m)\cdot V_s\cdot(W/HP)=B$	447	608	835	1,230	1,564	1,909
	$B^{0.80}$	132.0	169.0	217.7	296.5	359	422
	HP_{ao}/HP_{ro}	3.96	3.07	2.42	1.76	1.455	1.256
	$K_4=(HP_{ao}/HP_{ro})\,B^{0.80}\div\frac{L}{D}$	60.7	60.2	61.2	60.7	60.7	61.5
71	$(1/\eta_m)\cdot V_s\cdot(W/HP)=B$	526	706	981	1,441	1,838	2,255
	$B^{0.80}$	150.2	190.3	247.3	336.5	409	481
	HP_{ao}/HP_{ro}	3.54	2.81	2.15	1.575	1.306	1.155
	$K_4=(HP_{ao}/HP_{ro})\,B^{0.80}\div\frac{L}{D}$	61.6	62.1	61.6	61.5	62.0	64.4

Average $K_4=61.7$.

TABLE XX.

COMPARISON OF OBSERVED ABSOLUTE CEILING WITH THAT CALCULATED FROM EQUATION

$$\frac{HP_{ao}}{HP_{ro}}=\frac{61.7\frac{L}{D}}{\left(\frac{1}{\eta_m}\cdot V_s\cdot\frac{W}{HP}\right)^{0.80}}$$

Airplane.	V_s	$\frac{W}{HP}$	$\frac{1}{\eta_m}$	F	$F^{0.80}$	$\frac{L}{D}$	$\frac{HP_{ao}}{HP_{ro}}$	Absolute ceiling. From formula.	Absolute ceiling. Actual.
USXBIA	54	9.98	0.79	682	186	8	2.65	19,800	22,400
MB 3	58	7.00	.76	534	152	8	3.25	23,500	24,900
M 80	63	8.80	.78	711	192	8	2.57	19,300	19,900
"D"	55	8.10	.75	593	165	8	2.99	22,000	23,600
S 6	45	17.60	.805	984	234	8	2.11	15,700	15,100
Roland D VI-B	56	9.94	.78	713	193	8	2.57	19,300	19,000
MS-AR	51	17.60	.80	1,120	237	8	2.08	15,400	16,600
DH-4	62	10.20	.79	802	206	8	2.39	18,000	17,600
Fokker D VIII	57	9.00	.79	657	180	8	2.74	20,400	22,100
VE-7	52	11.60	.79	765	202	8	2.44	18,300	19,000
SE-5	54	11.40	.79	780	204	8	2.42	18,200	19,800
JN-4H	42	14.30	.75	802	205	8	2.42	18,200	19,000